Entropy-partition of Complex Systems and Emergence of Human Brain's Consciousness

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Abstract. The three frameworks for theories of consciousness taken most seriously by neuroscientists are that consciousness is a biological state of the brain,the global workspace perspective,and the perspective of higher state. Consciousness is discussed from viewpoint of theory of Entropy—partition of complex system in present article. Human brain's system self-organizably and adaptively implements partition 、 aggregation and integration, and consciousness emerges.The Gibss representation of consciousness is proved and That consciousness originates from quantum mechanical processes of brain activity is explained by means of SW entropy.

1.Intrduction

The three frameworks for theories of consciousness taken most seriously by neuroscientists are that consciousness is a biological state of the brain ,the global workspace perspective,and the perspective of higher state.In particular,in the article of Giulio Tononi and G.M. Edelman[1], studying of consciousness is concentrated on description of kinds of neural processes which can account for key properties of conscious experience. They have emphasized two key properties of consciousness: conscious experience is integrated (each conscious scene is unified) and at the same time it is highly differentiated (within a short time, one can experience any of a huge number of different conscious states), and have proposed notions so-called functional clustering, neural complexity and dynamical core hypothesis. Those researching methods and train of thought to problem of consciousness agree to a certain extent, with theory and method of Entropy-partition

of Complex System I proposed 31 years ago [2]. At that time, related specialists (appraising group of achievements in scientific research) ever have pointed out: theory of Entropy-partition (Aggregation) of complex system not only has been applied to Ecologico-Economical regionalization [3] of Ecologico-Economico-social complex system but also will certainly be applied to researching of human Brain's neural system. Therefore having read the article of G. Tononi and G.M. Edelman, I have much sensation seem to have met before, and excited feeling arises spontaneously. Though such, we strongly have felt it still necessary to discuss problem of consciousness from viewpoint of theory of entropy-partition (aggregation) of complex system.

2. Entropy-partition (aggregation) of neural system X

Suppose neural system $X = (X_1, X_2, \cdots, X_a, \cdots, X_p)^T$ is consisted of p neuron, $p \in N$ (N set of natural number), where $X_a = (X_{a_i})$, $a = 1, 2, \cdots, p$; $i = 1, 2, \cdots, q$. Let $C_a(a = 1, \cdots, p)$ be set of classification of X_a , $C_{a_i} = i$ be i-th element of C_a , then we have $C_a = \{1, 2, \cdots, i \cdots, k\}$, $k \leq q$, and let n_i be quantity for X_a belong to i-th class, then entropy of X_a is defined as

$$H(X_a) = -\sum_{i=1}^{k} n_i / q \log n_i / q \tag{1}$$

joint entropy of X_a, X_b is similarly defined as

$$H(X_a \bigcup X_b) = -\sum_i \sum_j n_{ij} / q \log n_{ij} / q \tag{2}$$

where n_{ij} is quantity for X_a belong to i-th class of C_a simultaneously X_b belong to j-th class of C_b . For the convenience of application, expressions (1) and (2) can respectively be represented as

$$H(X_a) = \log q - \frac{1}{q} \sum_{i=1}^{k} n_i \log n_i \tag{1'}$$

$$H(X_a \bigcup X_b) = \log q - \frac{1}{q}\sum_i \sum_j n_{ij} \log n_{ij} \qquad (2')$$

Having had above-mentioned definition of entropy, in what follows, correlative measure by which statistical dependence between the X_a and the X_b is denoted is defined by their mutual information.

Definition 1. Suppose $X_a \bigcap X_b = \phi$, then entropy

$$H(X_a, X_b) = H(X_a) + H(X_b) - H(X_a \bigcup X_b) \qquad (3)$$

is called correlative measure $\mu(X_a, X_b)$ between the X_a and the X_b.

Definition 2. Suppose $X_a \bigcap X_b = \phi$ for arbitrary $a, b(a \neq b)$, then

$$\mu(X_1, X_2, \cdots, X_p) \overset{\Delta}{=} \sum_{a=1}^{p} H(X_a) - H\left(\sum_{a=1}^{p} X_a\right) \qquad (4)$$

is called correlative measure among X_1, X_2, …, and X_p.

Definition 2′. Suppose system X be partitioned into m subsystems $s_1, s_2, \cdots, s_m$, for arbitrary i, j $(i \neq j)$, $s_i \bigcap s_j = \phi$, $X = \sum_{i=1}^{m} s_i$, then

$$\mu(s_1, s_2, \cdots, s_m) \overset{\Delta}{=} \sum_{i=1}^{m} H(s_i) - H\left(\sum_{i=1}^{m} s_i\right) \qquad (5)$$

is called correlative measure among $s_1, s_2, \cdots, s_m$.

Let us consider nonempty finite set X and set-family $E(X)$ consisted of its subset, P_e a set-function defined on $E(X)$ with properties

(i) $P_e(A) \geq 0$, $\forall A \in E(X)$,

(ii) $P_e(\phi) = 0$

Definition 3. If for arbitrary nonempty finite set $S_i \in E(X)$, $S_j \in E(X)$, $i \neq j$, $S_i \bigcap S_j = \phi$, have

$$P_e(S_i \bigcup S_j) \geq P_e(S_i) + P_e(S_j) \qquad (6)$$

then, set-function P_e satisfied conditions (i),(ii) is called superadditive.

Although finite superadditivities of mutual information or the so-called measure of cohesion of the components of the set of entities is well known result, it is not still unnecessary to write down our proof about finite, in particular, countably superadditivity of the correlative measure among some subsystems of the system X which is given.

Theorem[2]. Correlative measure $\mu(s_1, s_2, \cdots, s_m)$ is finitely superadditive, countably superadditive and unique.

Proof. ***Finite superadditivity.***

Suppose system X is partitioned into m subsystems s_1, s_2, ... , s_m, and that $s_i \in R, s_i \neq \varnothing; s_j \in R, s_j \neq \varnothing; s_i \bigcap s_j = \varnothing$, $X = \sum_{j=1}^{m} s_j \in R$, for any $i, j(i \neq j)$, where R is a algebra on set X which is given. By definition of the correlative measure, we have

$$\mu(S) = \mu(\sum_{j=1}^{m} s_j) = \mu(s_1, s_2, ..., s_m) = \mu(X_1, X_2, ... X_p) = \sum_{i=1}^{p} H(X_i) - H(\sum_{i=1}^{p} X_i)4 \quad (7)$$

$$
\begin{aligned}
\sum_{s_j \in S} \mu(s_j) &= \sum_{s_j \in S} (\sum_{X_j \in s_j} H(X_j) - H(\sum_{X_j \in s_j} X_j)) \\
&= \sum_{i=1}^{p} H(X_i) - \sum_{s_j \in S} H(s_j) \\
&= \sum_{i=1}^{p} H(X_i) - \sum_{j=1}^{m} H(s_j)
\end{aligned}
\quad (8)
$$

Subtracting (8) from (7) leads to

$$
\begin{aligned}
\mu(\sum_{j=1}^{m} s_j) - \sum_{j=1}^{m} \mu(s_j) &= \sum_{j=1}^{m} H(s_j) - H(\sum_{i=1}^{p} X_i) \\
&= \sum_{j=1}^{m} H(s_j) - H(\sum_{j=1}^{m} s_j) \geq 0
\end{aligned}
\quad (9)
$$

Countable superadditivity and uniqueness:

Suppose system X is partitioned into a sequence of subsystems $s_1, s_2, ..., s_n...$, $s_n \in R$ — algebra on X , $s_n \in R$, $s_n \neq \varnothing; s_m \in R, s_m \neq \varnothing; s_n \bigcap s_m = \varnothing$, $X = \sum_{n=1}^{\infty} s_n \in R$, for any $n, m(n \neq m)$.

Let $E_k = \sum_{n=1}^{k} s_n$, obviously, we have

$$E_1 \subset E_2 \subset \cdots \subset E_k \cdots, E_k \in R, \lim_{k \to \infty} E_k = \sum_{n=1}^{\infty} s_n \in R \qquad (10)$$

From nonnegativity and monotonicity of μ, we have

$$\mu(E_1) \leq \mu(E_2) \leq \cdots \leq \mu(E_k) \leq \cdots \leq \mu(\sum_{n=1}^{\infty} s_n) \qquad (11)$$

If $\lim_{k \to \infty} \mu(E_k) \to \infty$, from monotonicity of μ, then inevitably we have $\mu(\sum_{n=1}^{\infty} s_n) = \infty$. Hence by using finite superadditivity, we have

$$\mu(\sum_{n=1}^{\infty} s_n) = \infty = \lim_{k \to \infty} \mu(E_k) = \lim_{k \to \infty} \mu(\sum_{n=1}^{k} s_n) \geq \lim_{k \to \infty} \sum_{n=1}^{k} \mu(s_n) = \sum_{n=1}^{\infty} \mu(s_n)$$

If $\lim_{k \to \infty} \mu(E_k) = c < \infty$, above-mentioned process of proof is understood easily. And either of $\lim_{k \to \infty} \mu(E_k) = c < \infty$ and $\lim_{k \to \infty} \mu(E_k) = \infty$ makes uniqueness of the limit to hold still, i.e.

$\lim_{k \to \infty} \mu(E_k) = \lim_{k \to \infty} \mu(\sum_{n=1}^{k} s_n)$ is unique. Hence $\mu(s_1, s_2, \ldots, s_m)$ is unique. Up to this point, proof of the theorem is completed.

From above-mentioned property of correlative measure, criterion by which lowest requirement of ideal partition is presented should be

(i) $\mu(s_i) > \mu(s_i, s_j)$, $\mu(s_j) > \mu(s_i, s_j)$

(ii) $\mu(s_i) > \mu(s_{i_1}) + \mu(s_{i_2})$, $\mu(s_j) > \mu(s_{j_1}) + \mu(s_{j_2})$,

for any $i, j (i \neq j)$, $s_i \cap s_j = \phi$. Here s_{i_l}, s_{j_l} $(l = 1,2)$ denotes partition of s_i and s_j, respectively. (i) denotes that correlative measure of any subsystem itself which is obtained by partition is larger than correlative measure between it and any subsystem. (ii) denotes that correlative measure of any subsystem which is obtained by partition possesses strictly superadditivity.

When number of characteristic variable of system X is very large, comprehensive observation for data is impossible. Even though the observation is possible, as obtaining complete data spends very long time and for other reason such that obtained data loses its available value. At this time, statistical method and theory can be applied to obtaining data and to analyzing problem.

Suppose $\boldsymbol{x}_a = (x_{a_1}, x_{a_2}, \cdots, x_{a_N})$ be index of quantity of characteristic variable X_a of the complex system X, data obtained by mean of random sampling is $\bar{\boldsymbol{x}}_a = (x_{a_1}, x_{a_2}, \cdots, x_{a_q})$, $\bar{\boldsymbol{x}}_a$ be a random sample from population X_a. Obviously any x_{a_i} necessarily is equal to some $x_{a_{\theta_i}}$, here θ_i $(i = 1, 2, \cdots, q)$ is number of i-th individual of $\boldsymbol{x}_a$. For any θ_i, we have probability

$$p\left\{x_{a_1} = x_{a_{\theta_1}}, x_{a_2} = x_{a_{\theta_2}}, \cdots, x_{a_q} = x_{a_{\theta_q}}\right\} = (N-q)!/N! \tag{12}$$

Corresponding to expressions (1′) and (2′), we have

$$H(\bar{\boldsymbol{x}}_a) = \frac{(n-q)!}{n!}\left[\log(n!q) - \log(n-q)! - \frac{1}{q}\sum_i n_i \log n_i\right] \tag{13}$$

$$H(\bar{\boldsymbol{x}}_a \cup \bar{\boldsymbol{x}}_b) = \frac{(N-q)!}{N!}\left[\log(N!q) - \log(N-q)! - \frac{1}{q}\sum_i\sum_j n_{ij} \log n_{ij}\right] \tag{14}$$

At this time, correlative measure of the system X is defined by way similar to the above-mentioned method. Actually, in order to obtain the partition of the complex system X, we often introduce coefficient of correlative measure $\mu_{ab} = \mu(X_a, X_b)/H(X_b)$ $(\mu_{ij} = \mu(s_i, s_j)/H(s_j)$, obviously, $\mu(a,b)(\mu_{ij})$ is between 0 and 1. Having computed μ_{ab} for all a, b, those X_a, X_b whose correlative measure is larger than other $\mu(a, l)$, $l \neq b$ or $\mu(r, b)$, $r \neq a$, are combined so as to accord with the preceding criterion (i)，(ii). Hence some corresponding subsystems s_i, $i = 1, \cdots, m$, i.e. some neural functional units are obtained.

From neurophysiological studies, elementary unit representing information in human brain is mutually coordinated clique of neuron; by combining mutual function, neurons with different properties carry out correlative activity, form various dynamic neural networks which correspond to various function of information.

Above-mentioned partition or aggregation of neurons is carried out in self-organizable and self-adaptive form; the partition or aggregation of neurons,and integration make human brain to emerge consciousness.

3. Emergence of consciousness

Suppose system X self-organizably and self-adaptively is partitioned into m subsystem. Correspondingly, there are m subspace $(\Omega_i, \mathfrak{I}_i, P_i)$, $i = 1, \cdots, m$, of probability-space $(\Omega, \mathfrak{I}, P)$, where Ω is space of configuration of system X. Let $(\Omega, \mathfrak{I}, P)$ be Radon measurable space, then for any $\mathfrak{I}_i$ there exists regular conditional probability $p(\cdot \mid \mathfrak{I}_i)$, obviously, $p(\cdot \mid \mathfrak{I}_i) = p(\cdot \mid \mathfrak{I}(s_i)) = p(\cdot \mid s_i)$. Hence, for any $A \in \mathfrak{I}$,

$$P(A) = \int_{\Omega \mid \mu} p(\cdot \mid s_i) dp(s_i) \tag{15}$$

and for any random variable η,

$$E(\eta) = \int_{\Omega \mid \mu} E(\eta \mid s_i) dp(s_i) \tag{16}$$

where $\Omega \mid \mu$ is quotient space induced by correlative measure μ on Ω. Let us define following two functions,

$$f_\varepsilon^{(1)}(t) = \begin{cases} \dfrac{P(A)}{\varepsilon}, & 0 \leq t \leq \varepsilon \\ 0, & \varepsilon < t \end{cases} \tag{17}$$

$$f_\varepsilon^{(2)}(t) = \begin{cases} \dfrac{E\eta}{\varepsilon}, & 0 \leq t \leq \varepsilon \\ 0, & \varepsilon < t \end{cases} \tag{18}$$

where ε is arbitrary positive number, t is time.

To be this, we eventually have conscious experience, that is $P(A)$ or $E(\eta)$ under some situations or $\lim_{\varepsilon \downarrow 0} f_\varepsilon^{(i)}(t)$, $i = 1,2$, under other situations.

When A or η comes from outside of X, or when A or η is caused by "gene" or "experience" which is in inside of X, all will emerge consciousness, and is emerged by system X self-organizably and self-adaptively to be partitioned (aggregated), then to be integrated in hundreds of milliseconds. $P(A)$ or $E\eta$ is the integrated process.

Proposition. Suppose the neural system $X = (X_1, X_2, \cdots, X_a, \cdots, X_p)^T$ is the Abstract neural automata[4], then the consciousness $P(A)$ is Gibbsian.

Proof. Because the neural system $X = (X_1, X_2, \cdots, X_a, \cdots, X_p)^T$ is the Abstract neural automata[4],then conditional probability $p(\cdot \mid \mathfrak{I}_i) = p(\cdot \mid \mathfrak{I}(s_i)) = p(\cdot \mid s_i)$ is Gibbsian , $P(A) = \int_{\Omega|\mu} p(\cdot \mid s_i) dp(s_i)$ is a convex combination of Gibbs measures,hence Gibbsian.

The proof is thus completed.

If there exists sequences of mutual information between any appointed two neurons and between any appointed two subsystems, and the sequences of the mutual information have nice asymptotical properties, that is, if there exists ergodic superadditive process for the sequence of mutual information in neural system X , then the system can implement self-organization and self-adaptation.

4.Consciousness originates from quantum mechanical processes

We have pointed out ever that **"concept"** **" consciousness"** is generated at particle(ion) level in the brain and is experienced at the level of the neural network [6]. In the human brain system,there must be a transition from the microscopic or submicroscopic level to the macroscopic level, that is,from the quantum to the classical level for consciousness to generate at the ion level pass to be experienced at the level of neural network.When does the quantum process approach this transition point ?SW entropy can answer this question.

Suppose that human neural system at the ion level is denoted by Q,while the human neural system at level of neural network is denoted by N. In Q system, given state ρ ,then SW entropy is defined as[7]

$$I[\rho] = -\int dz dz^* p(z, z^*) \log p(z, z^*) \tag{19}$$

Where $|z>$ is a coherent state, $p(z, z^*) = <z \mid \rho \mid z>$,probability density.Von Neumann(vN) entropy is defined as

$$S(\rho) = -tr(\rho \log \rho) \tag{20}$$

In the case of discrete spectrum.

The lower bound of $I(\rho)$ is determined by two inequalities

$$I(\rho) \geq S(\rho) \tag{21}$$

$$I(\rho) \geq 1 \qquad\qquad (22)$$

The latter is saturated by coherent states, while the former by the thermal states of system Q(harmonic oscillator) in the high temperature regime. More precisely, $I(\rho)=1$,if and only if the system Q is in the coherent states, while taking maximum by $I = S$ corresponds to the thermal states of system Q. From a mathematical point of view, both of these states are Gaussian and therefore Gibbs states [8].Hence we can see that when the quantity $I = S + 1$, the system Q will approach the transition point from quantum to classical level. In essence, as the Q system evolves into the Gibbs states ,it will approach the transition point. Then the neural system will begin the process of experiencing consciousness.

5.Remarks

(1) That the neural system X is self-organizably and self-adaptively partitioned into some subsystems is implemented by means of the correlative measure, i.e. mutual information, which does not touch upon change of location of space of neurons or subsystems. This is a "Internet" in the neural System X . The some subsystems form large-scale hierarchical intelligent system. On the hierarchical intelligent system, basic principle of the intelligent system holds [5]. The layer with high intelligence is dynamic core of the neural system X .

(2) Consciousness is some memory.

(3) Unconsciousness is also consciousness.

(4) Consciousness is conscious of consciousness.

(5) Nearly 3000 years ago, Chinese ancient learned men said consciousness is that Saint hears at silence, looks at immateriality, and firmly believed memory all is in brain.,

(6) Oneself-consciousness can only be experienced; it forever cannot be captured by oneself.

(7) Consciousness as one of the most fascinating neurological phenomena—spirit is represented by a probability function. Essence of consciousness is probabilistic. Higher order state of consciousness is the thought. By means of learning (including perception, memory), humnn brain produces concept. Before ideology emerged, the concept is highest result of human's brain, is intension of cognitive object. In the theory of abstract neural automata,the concept is limit Gibss probability measure[4]. Thought is mutual connection of concepts, mutual transform of concepts, important stage of all evolutionary process of cognition.

In particular, In this paper, Gibbsness's proof of consciousness is given directly.

(8)From basic principle of the intelligent system[5],i have already proved the existence of universal intelligence ,thus proving the existence of universal consciousness ,because the consciousness has existed already ,before the generation of the intelligence.

(9)I have already proved " concept "- " consciousness" is generated at particle(ion) level in the brain and is experienced at the level of the neurons[6].

(10)Having decreased the corelative measure, maligant development of internet and artificial intelligence finally will lead up to weakening of of consciousness of human brain in most people. Obvious example is that miss among people will weaken.

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